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Real-Time Gravity Gradiometer Utilization to Improve INS Accuracy

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This paper details a method for deriving approximate low-order filters for estimation of velocity error in an inertial navigation system. The filter inputs are obtained from two sources—a gravity gradiometer and some external velocity reference. It is the intent here to show that these approximate filters are near-optimal. Covariance comparisons are used to evaluate filter performance. The first of these comparisons reveals that the approximate filter gives very nearly the same rms estimate error as the Kalman filter. This result, however, assumes a *rational* gravity perturbation model which, although unrealistic, is required to implement the Kalman filter. In addition, a multiple-measurement scheme employing the external velocity reference appears to significantly improve estimation accuracy. A second covariance study compares the rms estimation accuracy of the approximate filters obtained analytically and numerically when an irrational but more realistic gravity perturbation model is assumed. Agreement of these numbers justifies the "Schuler Dominance" assumption used in all the filter derivations and therefore gives greater weight to the claim of filter near-optimality.

Introduction

As instruments used in Inertial Navigation Systems (INS) have gotten better, the relative importance of gravity errors has increased. If a gravity gradiometer, which supplies information on the gravity vector, is coupled to an INS, it is possible to obtain an improved estimate of vehicle velocity and position. Because velocity error is the dominant error for some missions, this paper will focus on velocity error and its estimation.

Error Equations and Associated Gravity Perturbation Models

In order to analyze the error present in an INS, it is necessary to have the error equations associated with such a system. The complete error equations for an altitude and velocity damped, local level wander azimuth INS, coordinatized in a local level north pointing frame are given by Heller.[1] In Heller's report,[2] the single-axis expressions with no cross coupling are derived by assuming that the carrier vehicle moves in a straight line, there is no gyro drift, and the Earth is nonrotating. For purposes of this paper, the accelerometer bias will also be set equal to zero resulting in the following in-track error equations:

$$\delta \dot{R}_x = \delta V_x \tag{1}$$

$$\delta \dot{V}_x = -(g/R)\delta R_x - k(\delta V_x - w_2) + G_x \tag{2}$$

where δR_x is the in-track position error, δV_x is the in-track velocity error, g is the nominal pull of gravity at sea level, R is the radius of the Earth, k is the Schuler damping constant, w_2 is the error in external velocity measurement (assumed to be white noise), and G_x is the in-track gravity perturbation.

For the covariance analyses to follow, one of two statistical models for G_x will be used. The first is detailed in Appendix A of Ref. 2. To summarize briefly, it is a second-order model consisting of white noise put through a shaping filter as shown in Fig. 1. The parameter β in this figure is a Markov parameter dependent on vehicle velocity through the relation

$$\beta \triangleq V_x/D \times 2.146 \tag{3}$$

where V_x is the in-track vehicle velocity and D the correlation distance of the worldwide vertical deflection ensemble which is equal to 20 n.mi. The Laplace transform of the in-track gravity perturbation can be written as

$$G_x = \frac{1}{(s+\beta)^2} \cdot w_1 \tag{4}$$

The power spectrum of G_x is therefore

$$\Phi^T_{G_x}(s=j\omega_x) = \frac{1}{(s+\beta)^2} \cdot q_1 \cdot \frac{1}{(-s+\beta)^2} = \frac{q_1}{(-s^2+\beta^2)^2} \tag{5}$$

where q_1 is the magnitude of the power spectrum of w_1 in Eq. (4). The superscript T on the left-hand side of Eq. (5) indicates a spectral density based on time rather than distance. Stated more simply, ω_x has units of radians per unit time, not distance.

The second model to be used in the covariance analysis is more complex (but more realistic). This model, developed by Warren Heller, assumes several independent layers below the Earth's surface of "white noise" potential variations $U_{W_i}(i=1\text{-}5)$. The power spectrum of the potential fluctuations at an altitude z is related to the power spectrum of the potential fluctuations at sea level by

$$\Phi^S_U(\omega_x,\omega_y;z) = \sum_i e^{-2z_i\omega}\Phi_i = e^{-2\omega z}\Phi^S_U[\omega_x,\omega_y;z=0] \tag{6}$$

where $z_i = D_i + z$, D_i is the depth of the ith layer, $\omega = [\omega_x^2+\omega_y^2]^{1/2}$, and Φ_i is the spectral density of the potential variations on the ith level.

Heller proposes five separate layers, but only the three shallowest layers contribute appreciably to the short wavelength fluctuations. Table 1 gives the depths D_i and the spectral densities Φ_i for these three layers.

The two-dimensional power spectrum of G_x may be obtained from Eq. (6) by multiplying by ω_x^2

$$\Phi^S_{G_x}(\omega_x,\omega_y;z) = \omega_x^2 \sum_i \Phi_i e^{-2z_i\omega} \tag{7}$$

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To perform the covariance analysis, the one-dimensional power spectrum along the in-track direction is required. This is obtained as follows:

$$\Phi^S_{G_x}(\omega_x;z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi^S_{G_x}(\omega_x,\omega_y;z)\,d\omega_y \tag{8}$$

Carrying out this operation,

$$\Phi^S_{G_x}(\omega_x;z) = \frac{1}{\pi}\sum_{i=1}^{3}\Phi_i\left(\frac{\Omega}{2z_i}\right)^3 k_1(\Omega_i) \tag{9}$$

where

$$\Omega_i = 2z_i\omega_x \tag{10}$$

$$k_1(\Omega_i) = \int_0^{\infty} e^{-\Omega_i \cosh u}\cosh u\, du \tag{11}$$

Equation (11) is a modified Bessel function of first order.

Equation (9) gives the spatial-based spectrum $\Phi^S_{G_x}$. The time-based spectrum will be required. The spatial and time variation are related by the in-track velocity as given below.

$$\Phi^T_{G_x}(\omega_x;z) = (1/V_x)\Phi^S_{G_x}(\omega_x;z) \tag{12}$$

Note that, unlike the first model, Heller's model leads to a transcendental spectral density so that a conventional Kalman filter is not possible.

Kalman Covariance Analysis for the Second-Order Gravity Model

The INS/gradiometer configuration used to analyze the estimation errors is shown in Fig. 2. The filter shown in this schematic will be derived in this section using the Kalman filter theory, and in the next section using the Wiener filter theory.

Putting Eqs. (1), (2), and (4) into state space notation,

$$\overset{\dot{X}}{\begin{bmatrix}\dot{x}_1\\ \dot{G}_x\\ \delta\dot{V}_x\\ \delta\dot{R}_x\end{bmatrix}} = \overset{F}{\begin{bmatrix}-\beta & 0 & 0 & 0\\ 1 & -\beta & 0 & 0\\ 0 & 1 & -k & -g/R\\ 0 & 0 & 1 & 0\end{bmatrix}}\overset{X}{\begin{bmatrix}x_1\\ G_x\\ \delta V_x\\ \delta R_x\end{bmatrix}} + \overset{M}{\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & k\\ 0 & 0\end{bmatrix}}\overset{\bar{\omega}}{\begin{bmatrix}w_1\\ w_2\end{bmatrix}} \tag{13}$$

with x_1 being a dummy variable used to express Eq. (4) in state-space notation.

Assuming that the gradiometer furnishes a measurement of $\dot{G}_x$, the measurement equation is

$$z_1 = \overset{H}{[1\ \ -\beta\ \ 0\ \ 0]}\overset{X}{\begin{bmatrix}x_1\\ G_x\\ \delta V_x\\ \delta R_x\end{bmatrix}} + \overset{n}{(\delta\gamma_{xx}V)} \tag{14}$$

where $\delta\gamma_{xx}$ is the white noise gradiometer error, and V is the nominal in-track vehicle velocity. If the external velocity measurement is included as a Kalman filter input in the form

$$z_2 = (V_x)_{\text{computed}} - (V_x)_{\text{measured}} = \delta V_x - w_2$$

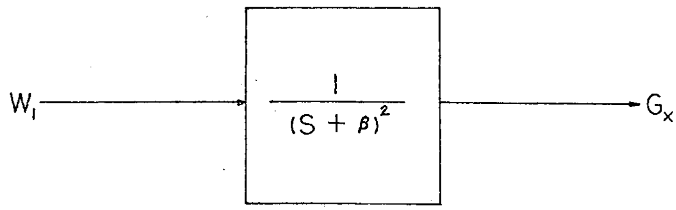

Fig. 1 Shaping filter.

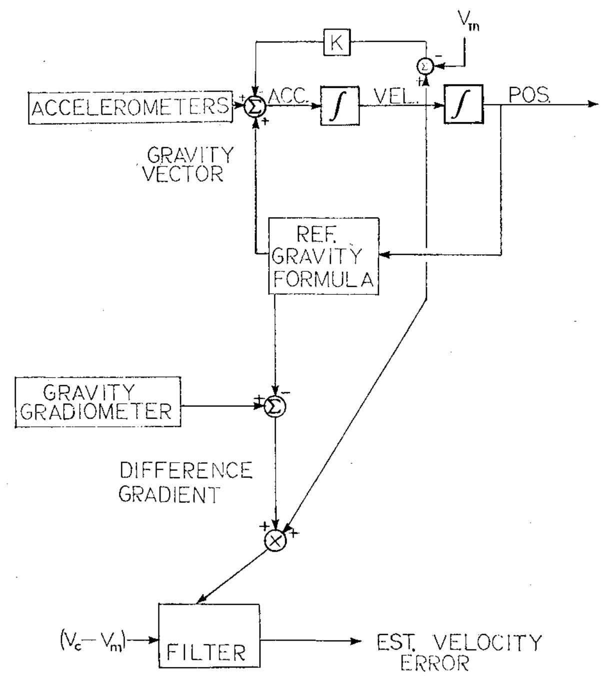

Fig. 2 INS with gradiometer.

Table 1 Heller's gravity perturbation model parameters

i	D_i, km	φ_i, km^6/s^4
1	16.3	7.13×10^{-8}
2	92.5	1.07×10^{-4}
3	390.5	1.16×10^{-2}

then

$$z_2 = \overset{H}{[0\ \ 0\ \ +1\ \ 0]}\overset{X}{\begin{bmatrix}x_1\\ G_x\\ \delta V_x\\ \delta R_x\end{bmatrix}} - w_2 \tag{15}$$

Equations (14) and (15) may be combined for a two-input Kalman filter as follows:

$$\overset{\bar{z}}{\begin{bmatrix}z_1\\ z_2\end{bmatrix}} = \overset{H}{\begin{bmatrix}1 & -\beta & 0 & 0\\ 0 & 0 & +1 & 0\end{bmatrix}}\overset{X}{\begin{bmatrix}x_1\\ G_x\\ \delta V_x\\ \delta R_x\end{bmatrix}} + \begin{bmatrix}n\\ -w_2\end{bmatrix} \tag{16}$$

It is obvious that in Eqs. (15) and (16) the measurement noise is correlated with the process noise. An equivalent system, whereby the process and measurement noises are not correlated, can be contrived however, using the technique presented in Ref. 3. Consequently, the conventional Kalman filter covariance analysis can be carried out.

Following the conventions set in Ref. 2, let w_2 be 0.2% of the actual velocity, with a 2-s correlation time. Then the spectral density of w_2 can be approximated by

$$q_2 = 4\cdot(0.002V)^2 \tag{17}$$

Also from Ref. 2,

$$q_1 = 4\beta^3 g^2 \sigma_\xi^2 \tag{18}$$

where σ_ξ^2 is the standard deviation of the worldwide vertical deflection ensemble (about 8 arc-sec) and

$$r = E[(\delta\gamma_{xx}V)^2]\cdot T \tag{19}$$

where r is the power spectrum of $\dot{G}_x$, and T is an assumed 10-s gradiometer averaging time.

To perform a steady-state Kalman filter covariance analysis on the system given by Eqs. (13-19), the computer program OPTSYS, in use at Stanford University, was employed. The results of this study, for a carrier vehicle velocity of 100 knots, are shown in Fig. 3, both for the single-input (z_1) filter and the two-input (z_1, z_2) filter. In this figure the δV_x estimate error (in mm/s) is plotted against gradiometer error (in Eotvos). From this graph, note the marked improvement in estimation accuracy when the external velocity measurement is included as a Kalman filter input.

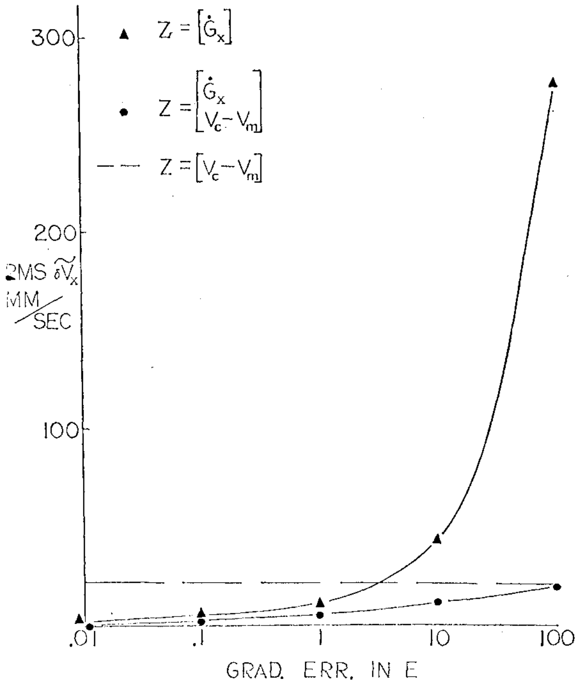

Fig. 3 Kalman filter accuracy.

Table 2 Estimate error power spectrum and covariance formulas for general gravity perturbation model

$\phi_{\delta V_x}(\omega)$	$\sigma^2_{\delta V_x}$
$\dfrac{(A-1)^2\omega^2\phi_{G_x}(\omega)+k^2\omega^2 q_2 + A^2 r}{[(\omega_s^2-\omega^2)^2+\omega^2k^2]}$	$\dfrac{r\phi_{G_x}(j\omega_s)}{2k[r+\omega_s^2\phi_{G_x}(j\omega_s)]}+\dfrac{kq_2}{2}$
$z' = \dot{G}_x$	
$\dfrac{(A-1)^2\omega^2\phi_{G_x}(\omega)+k'^2\omega^2 q_2 + A^2 r}{[(\omega_s^2-\omega^2)^2+k'^2\omega^2]}$	$q_2 k'$
$z' = \begin{bmatrix}\dot{G}_x\\ \delta V_x\end{bmatrix}$	

Wiener Filter Theory and Approximately Optimum Filters

An alternate method for deriving an optimal filter for velocity error estimation is a process known as the "Wiener-Hopf" technique. This method begins by assuming that

$$\underset{1\times 1}{\hat{y}} = \underset{1\times m}{\Psi^T(s)}\ \underset{m\times 1}{\bar{z}(s)} \tag{20a}$$

where $\hat{y}$ is the filtered estimate of some stationary process y based on a noisy vector measurement z of dimension m. $\Psi(s)$ is a transfer function with only left half plane (LHP) poles. Let

$$\underset{m\times 1}{\bar{z}} = \underset{m\times 1}{\bar{z}'} + \underset{m\times 1}{\bar{n}} \tag{20b}$$

with $\bar{n}$ being an m-dimensional white noise measurement error uncorrelated with the process y. Then the estimation error is

$$\tilde{y} = \hat{y} - y = \Psi^T(s)(\bar{z}' + \bar{n}) - y \tag{21}$$

with mean-squared error

$$\sigma_{\tilde{y}}^2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\Phi_{\tilde{y}}(s)\,\mathrm{d}s \tag{22}$$

in which

$$\Phi_{\tilde{y}}(s) = \Psi^T(-s)[\Phi_{z'z'}(s)+\Phi_n]\Psi(s) - \Psi^T(-s)\Phi_{z'y}(s)$$
$$-\Phi_{z'y}(-s)\Psi(s) + \Phi_{yy}(s) \tag{23}$$

where $\Phi_{yy}(j\omega_x)$ is the spectral density of y, the Fourier transform of its autocorrelation function (ACF). $\Phi_{z'z'}$ and $\Phi_{z'y}$ are the transforms of the ACF of the vector z' and of its cross correlation with y. The optimum $\Psi(s)$ is determined by the requirements that 1) $\sigma_{\tilde{y}}^2$ in Eq. (21) must be finite and 2) $[\Phi_{z'z'}(s)+\Phi_n]\Psi(s) - \Phi_{-z'y}(s)$ has no LHP poles. Condition 2 assures that any change in $\Psi(s)$ produces an increase in $\sigma_{\tilde{y}}^2$. The minimum mean-squared estimation error is then

$$\sigma_{\tilde{y}}^2 = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty}\{\Phi_{yy}(s) - \Phi_{z'y}(-s)\Psi(s)\}\mathrm{d}s \tag{24}$$

A much simplified approximate analysis results assuming that the Schuler frequency is lightly damped $(k \ll \omega_s)$ and if only the two LHP poles close to Schuler frequency are considered in the determination of $\Psi(s)$ and the evaluation of Eq. (24). This analysis is shown below for velocity error estimation, corresponding to the one- and two-measurement combinations discussed earlier. By assuming a general gravity perturbation model in these developments, the results can be interpreted with respect to either the second-order model or Heller's model.

A. Approximate Wiener Filter (One Measurement)

In the nomenclature of Eqs. (20a) and (20b), assume that

$$z = sG_x + n = z' + n \tag{25}$$

and, from the Laplace transform of Eqs. (1) and (2), that

$$y=\delta V_x=sG_x/(+)+skw_2/(+) \tag{26}$$

where

$$(+)=s^2+ks+g/R=s^2+ks+\omega_s^2$$

Hence, to determine $\Psi(s)$, the expression

$$-\Psi(s)s^2\Phi^T_{G_x}(s)+s^2/(+)\cdot\Phi^T_{G_x}(s)+\Psi(s)r \tag{27}$$

must have no LHP poles. Keeping only the obvious poles at $(+)=0$,

$$\Psi(s)\cong A/(+) \tag{28}$$

a simple second-order filter, where

$$A=\frac{\omega_s^2\Phi^T_{G_x}(j\omega_s)}{r+\omega_s^2\Phi^T_{G_x}(j\omega_s)}$$

and ω_s is the Schuler frequency.

The formulas given in the first row of Table 2 show the estimate error power spectrum and variance when Eq. (28) is substituted into Eq. (23) and the variance Eq. (24) is evaluated using Cauchy's residue theorem, where

$$A=\frac{\phi_{G_x}(j\omega_s)\omega_s^2}{r+\omega_s^2\phi_{G_x}(j\omega_s)} \qquad k'^2=(r/\omega_s^2q_2)A$$

B. Approximate Wiener Filter (Two Measurements)

For the case of two measurements

$$\tilde{z}=\begin{bmatrix} z_1 \\ z_2\end{bmatrix}=\begin{bmatrix} sG_x \\ \delta V_x\end{bmatrix}+\begin{bmatrix} n \\ -w_2\end{bmatrix} \tag{29}$$

and again

$$y=\delta V_x=(sG_x+skw_2)/(+) \tag{30}$$

Because z_2 is correlated with y, the previous theory is not directly applicable; however, a change of variables leads to a similar development. Assuming

$$\hat{y}=\Psi_1(s)z_1+\Psi_2(s)z_2$$

let y', z', $\phi(s)$ denote $sG_x/(+)$, sG_x, and $\Psi_2(s)-1$, respectively. Then $\tilde{y}=\hat{y}-y$ is expressible in the form:

$$\tilde{y}=\Psi_1(s)(z_1'+n)+\phi(s)(y'-h(s)w_2)-w_2 \tag{31}$$

where

$$h(s)=(s^2+\omega_s^2)/(+) \tag{32}$$

Hence

$$\Phi_{\tilde{y}}(s)=(\Psi_1(-s),\phi(-s))M(s)\begin{bmatrix}\Psi_1(s)\\ \phi(s)\end{bmatrix}$$
$$+q_2(\phi(-s)h(-s)+\phi(s)h(s)+1) \tag{33}$$

Table 3 Estimate error covariance formulas for second-order gravity perturbation model

$z'=\dot{G}_x$	$\sigma^2_{\delta\tilde{V}_x}=\dfrac{q_1}{2k[(\beta^2+\omega_s^2)^2+q_1/r\omega_s^2]}+\dfrac{kq_2}{2}$
$z'=\begin{bmatrix}\dot{G}_x\\ \delta V_x\end{bmatrix}$	$\sigma^2_{\delta\tilde{V}_x}=\left[\dfrac{q_1q_2}{[(\beta^2+\omega_s^2)^2+q_1/r\omega_s^2]}\right]^{1/2}$

where

$$M(s)=\begin{bmatrix} r-s^2\Phi^T_{G_x}(s) & -s^2\Phi^T_{G_x}(s)/(+) \\ \dfrac{-s^2\Phi^T_{G_x}(s)}{(-)} & \dfrac{-s\Phi^T_{G_x}(s)}{(+)(-)}+q_2\dfrac{(s^2+\omega_s^2)^2}{(+)(-)}\end{bmatrix} \tag{34}$$

in which $(-)$ denotes $(s^2-ks+\omega_s^2)$. Hence, the expression

$$M(s)\begin{bmatrix}\Psi_1(s)\\ \phi(s)\end{bmatrix}+\begin{bmatrix}0\\ q_2h(-s)\end{bmatrix} \tag{35}$$

must have no LHP poles, and the second term clearly has none. This condition, along with the conditions

$$\phi(s)\to-1 \quad \text{as} \quad s\to\infty \tag{36a}$$

$$s\Psi_1(s)\to 0 \quad \text{as} \quad s\to\infty \tag{36b}$$

determines $\Psi_1(s)$ and $\phi(s)$. All terms in the second column of $[M(s)]$ contain the denominator factor $(+)$. This means that $\phi(s)$ must be divisible by $(+)$. Furthermore, the poles of $\Psi_1(s)$ and $\phi(s)$ are the LHP poles of $|M(s)|=0$, since, if $\Psi_1(s)$ and $\phi(s)$ have residues A_{1j}, A_{2j} at s_j, then

$$M(s_j)\begin{bmatrix}A_{1j}\\ A_{2j}\end{bmatrix}=0$$

Therefore, the poles of $\Psi_1(s)$ and $\phi(s)$ must be LHP roots of the equation

$$|M(s)|=-\Phi^T_{G_x}(s)s^2[q_2(s^2+\omega_s^2)^2+r]+rq_2(s^2+\omega_s^2)^2=0 \tag{37}$$

If either

$$r/q_2\omega_s^4\ll 1 \tag{38a}$$

or

$$\Phi^T_{G_x}(j\omega_s)/q_2\omega_s^2\ll 1 \tag{38b}$$

then the dominant pole pair close to Schuler frequency is

$$s\cong-k'/2\pm j\omega_s \tag{39}$$

where

$$k'^2=\frac{\Phi^T_{G_x}(j\omega_s)r}{q_2[r+\omega_s^2\Phi^T_{G_x}(j\omega_s)]}$$

Because of condition (36a) and the divisibility of $\phi(s)$ by $(+)$, $\phi(s)$ is now determined to be (approximately)

$$\phi(s)\cong-(+)/(+') \tag{40}$$

where

$$(+')=s^2+k's+\omega_s^2$$

Then

$$\Psi_2(s)=1+\phi(s)\cong(k'-k)s/(+') \tag{41}$$

$\Psi_1(s)$ can be found by substituting Eq. (41) into the first row of Eq. (35)

$$\Psi_1(s)[-s^2\Phi^T_{G_x}(s)+r]+s^2\Phi^T_{G_x}(s)/(+') \tag{42}$$

which again must not have any LHP poles. Keeping only the obvious poles at $(+')=0$,

$$\Psi_I(s) \cong A/(+') \qquad (43)$$

By using the second-order filters given by Eqs. (41) and (43), the estimate error power spectrum can be found. From Eqs. (33) and (35), it follows that the minimum mean-squared estimation error is

$$\sigma_{\tilde{y}}^2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} q_2(1+h(s)\phi(s))\mathrm{d}s$$

$$\cong \frac{q_2}{2\pi j}\int_{-j\infty}^{j\infty} \frac{k's}{(+')}\mathrm{d}s$$

$$= q_2 k' \qquad (44)$$

The expressions for the estimate error power spectrum and the mean-squared estimation error are given in the second row of Table 2. It is interesting to note that the minimum over k of the one-measurement variance is identical to the two-measurement variance. This means that if the optimum k is used with only the gradiometer as a filter input, the estimate of δV_x is as accurate as though both the gradiometer measurement and external velocity had been supplied to the filter.

Comparison of Results for the Second-Order Model

Table 3 shows the results of substituting Eq. (5) for the general gravity model ($\Phi_{G_x}(s)$) in the preceding variance approximations.

In these formulas and those presented in Table 2, emphasis was put on having the estimate error power spectrum highly peaked at Schuler frequency. The validity of this assumption can be proven by comparing the analytic formulas derived from Eq. (24) with the estimate error variance obtained from OPTSYS. Table 4 compares the OPTSYS results with those from the analytic formulas given in Table 3 for an in-track velocity of 100 knots, a k of 4.5210^{-4} 1/s, and rms gradiometer errors between 1 and 100 Eotvos. Agreement is good except for the two measurement cases with high gradiometer error, in which condition (38a) is marginally satisfied.

Covariance Analysis for the Heller Model

The spectrum for the Heller model, given by Eqs. (9) and (12), is substituted for $\Phi_{G_x}^T(s)$ in the general approximations.

Table 4 Comparison of OPTSYS data (in mm/s) with analytically obtained approximate formulas

	$z'=\dot{G}_x$	$z'=\begin{bmatrix}\dot{G}_x\\ \delta V_x\end{bmatrix}$
$\delta^{AF\,a}_{\delta V_{x1E}}$	5.3527	5.196
$\sigma^{OPT\,b}_{\delta V_{x1E}}$	5.3525	5.118
$\sigma^{AF}_{\delta V_{x10E}}$	43.579	16.381
$\sigma^{OPT}_{\delta V_{x10E}}$	43.458	12.332
$\sigma^{AF}_{\delta V_{x100E}}$	293.606	42.5715
$\sigma^{OPT}_{\delta V_{x100E}}$	275.4	20.602

[a] AF = approximate formula.
[b] OPT = OPTSYS.

Table 5 Comparison of rms estimate (in mm/s) error obtained from approximate formulas and numerical integration using Heller's gravity perturbation model

	$z'=\dot{G}_x$	$z'=\begin{bmatrix}\dot{G}_x\\ \delta V_x\end{bmatrix}$
$\sigma^{AF\,a}_{\delta V_{x1E}}$	1.079×10^1	5.196
$\sigma^{NI\,b}_{\delta V_{x1E}}$	1.085×10^1	5.265
$\sigma^{AF}_{\delta V_{x10E}}$	1.059×10^2	1.634×10^1
$\sigma^{NI}_{\delta V_{x10E}}$	1.065×10^2	1.616×10^1
$\sigma^{AF}_{\delta V_{x100E}}$	5.895×10^2	3.854×10^1
$\sigma^{NI}_{\delta V_{x100E}}$	5.839×10^2	2.894×10^1

[a] AF = approximate formula.
[b] NI = numerical integration.

The resulting variances are shown, with superscript "AF" in Table 5 for various measurement error levels, for both one- and two-measurement filters. Again the two-measurement filter is superior. The companion numbers, labeled with superscript "NI" are obtained by numerical integration of the error spectrum:

$$\sigma_{\tilde{y}}^2 = \frac{1}{\pi}\int_0^\infty \Phi_{\tilde{y}}(j\omega_x)\mathrm{d}\omega_x$$

using formulas (23) or (33) with Heller's spectrum together with the approximately optimum second-order filters. The close agreement with the "AF" values is to be expected since the spectra were indeed peaked near Schuler frequency.

Conclusions

The Wiener-Hopf technique provides a method for determining a near-optimal filter for velocity error estimation even when the assumed gravity perturbation model is not a rational expression. In fact, this method can be used with any model with the limitation that the Schuler portion of the gravity perturbation be dominant in the δV_x estimate error spectrum. This method will also accommodate the use of other gravity-gradient components. The estimate error data presented suggest that the external velocity measurement significantly increases estimation accuracy when it is used as filter input. As pointed out earlier, however, this same increase in accuracy can be obtained through the use of an optimal value of k, the Schuler damping constant.

Acknowledgments

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